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The Condensation of Mixed Vapors

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Although condensation from saturated mixed vapors upon a cold surface has been frequently explored experimentally, current theories describing this phenomenon have not been adequately tested. This work describes the condensation process in which two condensable components are involved and how, consequently, the mass transfer process is involved in their partial fractionation.

A discussion of the Bedingfield correction (*3*) for the effect of molecular weight difference of the diffusing components upon the mass transfer coefficient leads to its necessary association with the Ackermann factor (*4, 5*) for the general mass transfer effect. Data from two sets of experiments using different apparatuses demonstrate the use of this net corrective factor and also demonstrate the general applicability of the Drew-Colburn mass transfer coefficient, for transfer processes in which more than one component transfers from one phase to another.

The fundamental description of condensation from turbulent mixtures of vapors was set forth by Colburn and Hougen (*6*) for a binary mixture of one condensable and one noncondensable vapor. The mass transfer aspects of this approach were further generalized by Colburn and Drew (*5*) to include two condensable vapors and with less quantitative merit to include more than two components. The Colburn-Drew mass transfer coefficient F, which resulted from the above analyses, was defined by the following equation:

$$\omega = F \ln \frac{z - y_v}{z - y_o} \qquad (1)$$

This coefficient is the same as the Stewart coefficient (*4*), although the defining equation is given in a different form.

For an adequate description of the condensation process, the above mass transfer equation is coupled with the heat transfer equation that is usually embodied as some form of the familiar Nusselt equation. The utility of this latter equation has been demonstrated experimentally and is well described in standard heat transfer texts (*11*).

Mass transfer considerations in the condensate phase further complicate the required analysis. This problem has been investigated experimentally by Kent and Pigford (*10*), who measured the changing vapor phase concentration as condensation continuously occurred along a vertical tube condenser. From the data of these investigators it is probably true that in ordinary condensation configurations the vapor phase offers the larger mass transfer resistance. Although the method for describing the diffusional process in the turbulent vapor stream from which more than one component condenses had been suggested in the early engineering literature, onlv Kent and Pigford have made use of this description.

The experiments described in this paper attempt to demonstrate directly the condensation process in which more than one component enters the liquid condensate phase.

THEORY

Equation (1) is derived from the effect of mass transfer upon the concentration profile in the vapor in the vicinity of the condensate surface. The fluid velocity profile in this region is assumed to be identical to that for which no mass transfer occurs; turbulence exists in the main body of flowing vapor and the model visualized is essentially that described by film theory. Bird, Stewart, and Lightfoot compared the effects of mass transfer as described by the penetration model and boundary layer theory also (*4*).

Equation (1) as written defines the coefficient F, which is independent of mass transfer within the limitations of film theory. However, to offer a complete discussion of any mass transfer process, its comparison with the equivalent heat transfer process must be made. Bedingfield and Drew (*3*) pointed out that when the diffusing species are of different molecular weights the convective velocity in the substantial derivative of the Fourier-Poisson equa-

Table 1. Values of Ackermann and Bedingfield Correction Factors

Run No.	β	$\dfrac{\beta a}{e^a - 1}$
M-2-5	0.307	0.863
M-4-1	0.595	0.827
M-5-1	0.366	2.29
M-5-2	0.366	1.52
M-5-4	0.366	1.36
M-5-5	0.366	1.01
M-5-6	0.366	1.45
M-5-8	0.366	1.11
M-6-1	0.450	0.837
M-7A-1	0.348	0.732
MM-1	0.566	0.890
MM-2	0.489	1.02
MM-3	0.467	1.01
MM-4	0.418	1.09
MM-5	0.628	0.892
MM-6	0.556	0.895

tion should be the molal-average velocity, since it is for this frame of reference that the binary diffusion coefficient is independent of concentration. The velocity in the corresponding heat transfer equation is the mass-average velocity. They show that the diffusion equation may be written in terms of the mass-average velocity instead of the molal-average velocity by using (ln M) as the dependent variable in place of the volumetric concentration. The equation in this form is directly equivalent to the corresponding Fourier-Poisson equation for heat transfer. By writing both equations in dimensionless form they become identical, and the solution for the heat transfer situation may be carried over directly to mass transfer within the limits of the assumptions of the original equation of change as written. This equation of change is

$$\frac{\partial X}{\partial \theta} + \vec{V} \cdot \nabla X = K \nabla^2 X$$

in which $K = D_v$, for mass transfer; $K = \dfrac{k}{\rho c_p}$, for heat transfer; $X = \dfrac{\ln M - \ln M_v}{\ln M_o - \ln M_v}$, for mass transfer; and $X = \dfrac{t - t_v}{t_o - t_v}$, for heat transfer. The solutions X to the above equation enter as the derivatives at the interface through their relationship with the heat transfer and mass transfer coefficients.

$$F = -\frac{D_v \rho_M}{D_o} \beta (\nabla X)_o$$

$$h = -\frac{k}{D_o} (\nabla X)_o$$

$$\beta = \frac{M_o \ln M_o/M_v}{(M_B - M_A)(z - y_o) \ln \dfrac{z - y_o}{z - y_v}} \qquad (2)$$

The corrective factor β comes about by the use of ln M in place of C_A. $(\nabla X)_o$ is the appropriate dimensionless derivative of the solution to the differential equation at the interface. Since $\dfrac{h D_o}{k}$ is known from experiment for a given geometry, so is $\dfrac{F D_o}{D_v \rho_M \beta}$ known by making proper substitutions of equivalent dimensionless groups. However, when mass transfer occurs the $(\nabla X)_o$ factor for heat transfer is not adequately described by its Reynolds number and Prandtl number-dependence, because the Ackermann corrective term is directly involved (see Bird, Stewart, and Lightfoot, page 658). This term is

$$\frac{a}{e^a - 1}$$

where

$$a = \frac{\omega C_{pz}}{h} \text{ (heat transfer)}; \quad a = \frac{\omega}{F} \text{ (mass transfer)} \qquad (3)$$

and h and F refer only to the coefficient when the mass transfer approaches zero. Therefore, the expressions for the coefficients should indicate the mass transfer dependence of the coefficients and should indicate the presence of a.*

$$\frac{h^{\bullet} D_o}{k} = -\nabla(X)_o = \psi(N_{Re}, N_{Pr}, a)$$

$$\frac{F^{\bullet} D_o}{D_v \rho_M \beta} = -\nabla(X)_o = \psi(N_{Re}, N_{Sc}, a)$$

Although the Ackermann corrective factor (*1*, *4*, *5*) has its justification in film theory, it has been shown by Mickley et al. (*12*) to hold in wind tunnel work over a large range of mass transfer rates, and so may be taken to be generally applicable. The last equation above may be written perhaps generally

$$\frac{F^{\bullet} D_o}{D_v \rho_M \beta} = \psi(N_{Re}, N_{Sc}) \frac{a}{e^a - 1}$$

and it is the product $\dfrac{\beta a}{e^a - 1}$, which is the net correction factor required to maintain the heat transfer-mass transfer analogy. Since the ratio of the a's (obtained from heat and from mass transfer definitions) is the ratio of Prandtl to Schmidt numbers to some exponent less than one for ordinary mass transfer-heat transfer processes, the two a's are almost numerically equal ordinarily.

RESULTS

Method I

These methods were obtained from studies of the condensation of acetone and toluene vapors in an annulus with transfer occurring to the inner surface. The experimental method is given in the Appendix.†

Figure 1 illustrates the mass transfer data with and without Bedingfield and Ackermann corrections. These data are compared to a plot of the Wiegand equation (*14*) for heat transfer from the inner surface of an annulus

$$\frac{h}{C_p G_f} N_{Pr}^{2/3} = 0.023 \left(\frac{D_2}{D_1}\right)^{0.45} N_{Re}^{-0.2}$$

Both the Reynolds number and the mass flow G_f were based on the velocity of the vapor relative to the velocity of the liquid surface as computed by the Nusselt equation for film flow.

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* $h^{\bullet}$ has the same significance here as it has in Bird, Stewart, and Lightfoot. However, the significance of $F^{\bullet}$ is not the same because of the introduction of the Bedingfield factor and the use of the Drew-Colburn coefficient rather than the Stewart coefficient.

† Deposited as document 8403 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $2.50 for photoprints or $1.75 for 35-mm. microfilm.

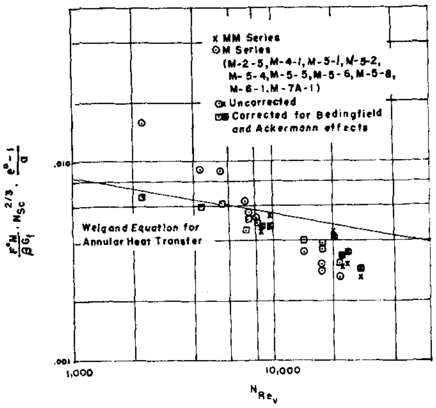

Fig. 1. Condensation of mixed vapors-mass transfer data, method I.

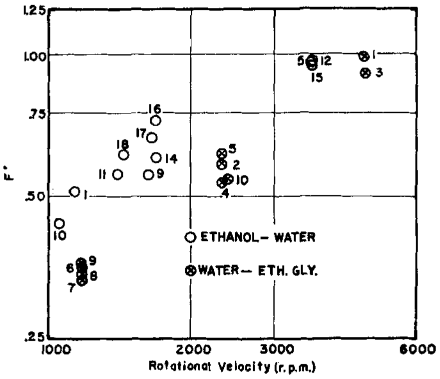

Fig. 2. Experimental coefficient vs. rotational velocity, method II.

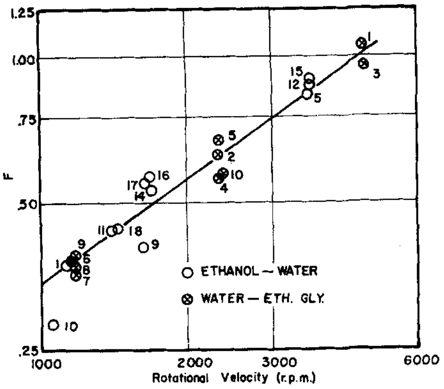

Fig. 3. Mass transfer coefficient vs. rotational velocity, method II.

It is evident on examination of Figure 1 that application of Ackermann and Bedingfield factors causes the data to fall closer to the predicted plot than the unmodified data. The probable error of these derived mass transfer coefficients has been evaluated by standard means as ± 30% (*8*). Values of β and $\frac{\beta a}{e^a - 1}$ are listed in Table 1.

Method II

The experimental technique (*9*) that yielded these results is discussed in the Appendix. Figures 2 through 5 refer to condensation from ethylene glycol-water and ethanol-water systems. This condensation occurred upon a horizontal layer of condensate continuously withdrawn from the surface to maintain a steady state. The concentration of the test condensate was therefore equal to z. This statement is justified by the flow pattern of the condensate layer being, for the most part, straight down so that diffusional effects in the liquid due to lateral flows were minimized. The effects of physical property differences for all systems investigated are not significant if a correlation such as that used by Johnson and Huang (*9*) is applicable. Therefore, these are not indicated in the discussion and results that follow.

Figure 2 illustrates a plot of the experimentally determined mass transfer coefficient vs. the rotational velocity. It is noted that the data from both systems do not determine any particular curve. If the physical property differences are not significant for the two, the results of Johnson and Huang (*9*) suggest that they should determine such a curve unless the rate of mass transfer is in some way involved.

The coefficient F corrected for mass transfer is plotted against rotational velocity in Figure 3. The data were used to compute F by means of the relationship

$$F = \frac{\omega}{\ln\left(1 + \frac{\beta\,\omega}{F^{\bullet}}\right)}$$

This equation was obtained as follows. From the results of Johnson and Huang (*9*) for low mass transfer rates, the coefficients should correlate with rotational velocity (N_{Re}) in the following way.

$$\frac{FD}{D_v\,\rho_M} = b\,(N_{Re})^{0.71}$$

When the vapor phase is involved and mass transfer rates are large, from the earlier analysis this equation should be written

$$\frac{1}{\beta}\left(\frac{F^{\bullet}D}{D_v\,\rho_M}\right) = b\,(N_{Re})^{0.71}\left(\frac{a}{e^a - 1}\right)$$

in which

$$a = \frac{\omega}{F} = \left(\frac{\omega}{b\,(N_{Re})^{0.71}}\right)\left(\frac{D}{D_v\,\rho_M}\right)$$

The above equations may be combined to give

$$1 + \frac{\beta\omega}{F^{\bullet}} = e^a$$

Figures 4 and 5 show this relationship for the two systems. The slopes through both sets of data are the same. These data have been given elsewhere.* Experiments were also carried out with Cellosolve and water. These results showed an apparent marked effect of mass transfer in the $F^{\bullet}$ rotational velocity plot. The $\ln(1 + \beta\omega/F^{\bullet})$ vs. $\omega/(n)^{.71}$ plot, however, correlated well but did not extrapolate through zero. Further investigation into this behavior revealed that the equilibrium data (*2*) were questionable. A systematic error in these literature data was shown to be one explanation for the strange behavior.

The data in Figures 4 and 5 contain ω in both ordinate and abscissa variables. This duplication is superficial for in fact the experimental value for ω appears only in the

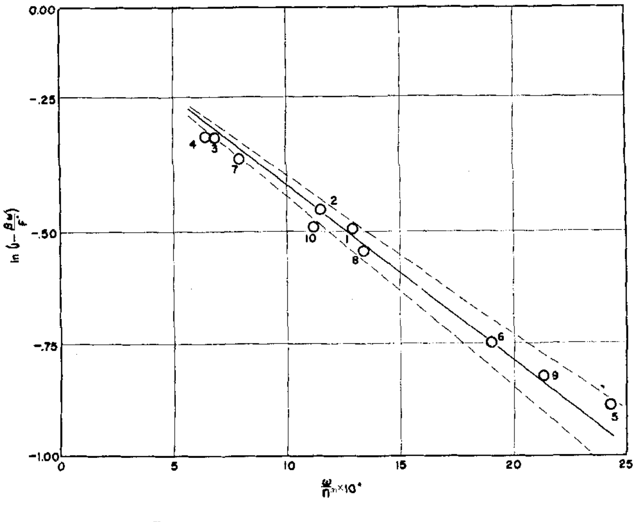

Fig. 4. Water-ethylene glycol system.

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* See footnote on page 801. Besides the Appendix, Tables 2 and 3 and Figures 6 to 10 are available.

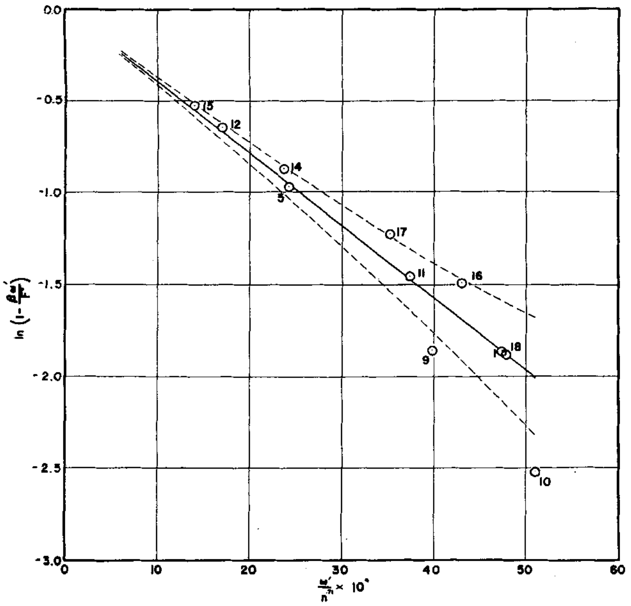

Fig. 5. Ethanol-water system.

abscissa; the term $\frac{\beta\omega}{F^\bullet}$ in the ordinate is equal to $-\frac{M_o \ln M_o/M_v}{(M_B - M_A)(z - y_o)}$ which is dependent only upon experimental concentration determinations and the equilibrium data. The dotted lines in Figures 4 and 5 locate 5% deviations in the values of $\frac{\beta\omega}{F^\bullet}$ contained in the solid straight line. In Figures 4 and 5 the axes are labeled ω', which is taken to be positive; that is $\omega' = -\omega$. [Customarily, transfer into the phase is taken as positive (*4*).]

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NOTATION

A = area for mass transfer, sq.ft.
a = Ackermann correction defined in Equation (3)
C_p = molal heat capacity at constant pressure, $\frac{\text{B.t.u.}}{(\text{lb. mole})(^\circ\text{R.})}$
c_p = heat capacity at constant pressure, $\frac{\text{B.t.u.}}{(\text{lb.})(^\circ\text{R.})}$
D_o = linear reference dimension, ft.
D_v = volumetric coefficient of diffusion, $\frac{\text{sq.ft.}}{\text{hr.}}$
D = diameter, ft.
F = mass transfer coefficient, $\frac{\text{lb. moles}}{(\text{hr.})(\text{sq.ft.})}$
G = mass velocity, $\frac{\text{lb.}}{(\text{hr.})(\text{sq.ft.})}$
h = heat transfer coefficient, $\frac{\text{B.t.u.}}{(\text{hr.})(\text{sq.ft.})(^\circ\text{F.})}$
k = thermal conductivity, $\frac{\text{B.t.u.}}{(\text{hr.})(\text{ft.})(^\circ\text{F.})}$
M = mean molecular weight, $\frac{\text{lb.}}{\text{lb. mole}}$
N_{Pr} = Prandtl number, $\frac{c_p\,\mu}{k}$
N_{Re} = Reynolds number = $\frac{D_o V \rho}{\mu}\left(\frac{4\Gamma}{\mu}\text{ for film flow}\right)$
N_{Sc} = Schmidt number = $\frac{\mu}{\rho D_v}$
t = temperature, °F.
W = total molal transfer rate, $\frac{\text{lb. moles}}{(\text{hr.})}$
x = distance along condenser from top, ft.
y = mole fraction in the vapor phase of component A
z = local molar transfer for component A divided by the total local molar transfer

Greek Letters

β = Bedingfield correction factor defined in Equation (2)
Γ = mass rate of liquid flow per unit periphery, $\frac{\text{lb.}}{(\text{hr.})(\text{ft.})}$
θ = time, hr.
λ = molal latent heat of vaporization
μ = viscosity, $\frac{\text{lb.}}{(\text{hr.})(\text{ft.})}$
ρ = mass density, $\frac{\text{lb.}}{\text{cu.ft.}}$
ρ_M = molal density, $\frac{\text{lb. moles}}{\text{cu.ft.}}$
ϕ = $\left[k\frac{(\rho^2 g)^{1/3}}{\mu^2}\right]_L$, $\frac{\text{B.t.u.}}{(\text{hr.})(\text{sq.ft.})(^\circ\text{F.})}$
ω = local molar transfer rate, $\frac{\text{lb. moles}}{(\text{hr.})(\text{sq.ft.})}$

Subscripts

L = liquid phase value
m = length mean value
o = interfacial value
v = bulk, vapor stream value
w = solid wall value

Superscripts

$\bullet$ = experimental value

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